evaluate the integrals using the indicated substitutions.

Week 20-7 (CH6, SEC 3) #2, 10, 24, 34

#2. (a) $\int \sec^2(4x+1)\,dx$; $u=4x+1$

$\sec^2 u\,du$ $\quad \frac{1}{4}\int \sec^2 u\,du = \frac{1}{4}\tan u + C$

$\to \frac{1}{4}\tan(4x+1) + C$

(b) $\int y\sqrt{1+2y^2}\,dy$; $u = 1+2y^2$

$\sqrt{u}\,du \Rightarrow \quad u^{\frac{1}{2}}\,du \quad \frac{1}{4}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{6}u^{\frac{3}{2}} + c \longrightarrow \frac{1}{6}(1+2y^2)^{\frac{3}{2}} + C$

(c) $\int \sqrt{\sin\pi\theta}\,\cos\pi\theta\,d\theta$; $u = \sin\pi\theta$

$\frac{1}{\pi}\int u^{\frac{1}{2}}\,du = \frac{2}{3\pi}u^{\frac{3}{2}} + C = \frac{2}{3\pi}\sin^{\frac{3}{2}}\pi\theta + C$

(d) $\int (2x+7)(x^2+7x+3)^{\frac{4}{5}}\,dx$; $u = x^2+7x+3$.

$\int u^{\frac{4}{5}}\,du = \frac{5}{9}u^{\frac{9}{5}} + C$

$\frac{5}{9}(x^2+7x+3)^{\frac{9}{5}} + C$.

#10. $\int x^3\sqrt{5+x^4}\,dx$ $\quad u = 5+x^4$

$\frac{1}{4}\int \sqrt{u}\,du = \frac{1}{6}u^{\frac{3}{2}} + C$.

$\frac{1}{6}(5+x^4)^{\frac{3}{2}} + C$.

#24. $\int \frac{\sin(1/x)}{3x^2}\,dx \qquad (u = \frac{1}{x})$

$du = -\frac{1}{x^2}\,dx$

$-\frac{1}{3}\int \sin u\,du = \frac{1}{3}\cos u + C = \frac{1}{3}\cos\left(\frac{1}{x}\right) + C$

#34. $\int \underline{\cos 2t}\ \sin^5 2t\,dt.$

$u = \sin 2t \quad du = 2x\,dx$

$\frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + C$

$\Rightarrow \frac{1}{12}\sin^6 2t + C$